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On the component structure of $\mathcal{N}=1$ supersymmetric nonlinear electrodynamics

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Abstract: We analyze the component structure of models for 4D $\mathcal{N}=1$ supersymmetric nonlinear electrodynamics that enjoy invariance under continuous duality rotations. The $\mathcal{N}=1$ supersymmetric Born-Infeld action is a member of this family. Such dynamical systems have a more complicated structure, especially in the presence of supergravity, as compared with well-studied effective supersymmetric theories containing at most two derivatives (including nonlinear Kähler sigma-models). As a result, when deriving their canonically normalized component actions, it becomes impractical and cumbersome to follow the traditional approach of (i) reducing to components; and then (ii) applying a field-dependent Weyl and local chiral transformation. It proves to be more efficient to follow the Kugo-Uehara scheme which consists of (i) extending the superfield theory to a super-Weyl invariant system; and then (ii) applying a plain component reduction along with imposing a suitable super-Weyl gauge condition. Here we implement this scheme to derive the bosonic action of self-dual supersymmetric electrodynamics coupled to the dilaton-axion chiral multiplet and a Kähler sigma-model. In the fermionic sector, the action contains higher derivative terms. In the globally supersymmetric case, a nonlinear field redefinition is explicitly constructed which eliminates all the higher derivative terms and brings the fermionic action to a one-parameter deformation of the Akulov-Volkov action for the Goldstino. The Akulov-Volkov action emerges, in particular, in the case of the $\mathcal{N}=1$ supersymmetric Born-Infeld action.

Keywords: Superspaces, Supersymmetry and Duality, Supersymmetric Effective Theories, Supergravity Models.

Contents

1. Introduction

The Born-Infeld theory [1] is a particular representative in the family of models for nonlinear electrodynamics which are grouped together through a single classification principle of self-duality, that is invariance under continuous electromagnetic duality rotations [2−4]. The requirement of self-duality is equivalent to the fact that the lagrangian $L(F)$ is a solution to the (non-supersymmetric) self-duality equation [3, 4]

$$\tilde{F}^{ab}\,F_{ab} + \tilde{G}^{ab}\,G_{ab} = 0\,, \qquad \tilde{G}^{ab}(F) \equiv \frac{1}{2}\varepsilon^{abcd}\,G_{cd}(F) = 2\,\frac{\partial L(F)}{\partial F_{ab}}\,,$$

with $\tilde{F}$ the Hodge-dual of F. What makes the Born-Infeld model unique is, in particular, its appearance as a low-energy effective action in string theory [5, 6]. It is worth mentioning that a general theory of (nonlinear) self-duality in four and higher space-time dimensions for non-supersymmetric theories was developed in [7].

The concept of self-dual nonlinear electrodynamics [3, 4] was extended to 4D $\mathcal{N} = 1,\ 2$ globally supersymmetric theories [8, 9]. Such a marriage of nonlinear electromagnetic self-duality with supersymmetry has turned out to be quite robust, since the families of actions obtained include all the known models for partial breaking of supersymmetry based on the use of a vector Goldstone multiplet. In particular, the $\mathcal{N} = 1$ supersymmetric Born-Infeld action [10], which is a Goldstone multiplet action for partial supersymmetry breakdown $\mathcal{N} = 2 \to \mathcal{N} = 1$ [11, 12], appears, at the same time, to be a solution to the $\mathcal{N} = 1$ self-duality equation [8, 9]. Furthermore, the model for partial breaking of supersymmetry $\mathcal{N} = 4 \to \mathcal{N} = 2$ developed in [13] proves to be a unique solution to the $\mathcal{N} = 2$ self-duality equation possessing a nonlinearly realized central charge symmetry [9].

Self-dual supersymmetric electrodynamics can naturally be coupled to superfield supergravity [14], using either the old minimal [15, 16] or the new minimal [17] formulations of $\mathcal{N} = 1$ supergravity (see textbooks [18 − 20] for reviews on superfield supergravity). As demonstrated in [14], such dynamical systems possess quite remarkable properties including (i) duality-invariance of the supercurrent; (ii) self-duality under Legendre transformation. These properties are a natural generalization of similar properties in the non-supersymmetric case [3, 4] and in the globally supersymmetric case [8, 9]. An unexpected feature of self-dual locally supersymmetric systems is that they couple not only to the dilaton-axion chiral multiplet (that transforms under duality rotations), but also to those nonlinear Kähler sigma-models which are inert under duality rotations.

While the considerations in [8, 9, 14] were given mainly in terms of superspace and superfields, here we would like to subject to scrutiny the component structure of self-dual supersymmetric systems. This turns out to involve two rather nontrivial aspects.

For general supergravity-matter systems with *at most two derivatives* at the component level [21, 22], the traditional approach (reviewed in [19]) of obtaining canonically normalized component actions consists of two steps: (i) a plain reduction from superfields to components; (ii) the application of a field-dependent Weyl and local chiral transformation (accompanied by a gravitino shift). Now, as we turn to nonlinear supersymmetric electrodynamics, a generic term in the component action may involve *any number of derivatives* — already the purely electromagnetic part of the lagrangian, $L(F)$, is a nonlinear function of the field strength. For such supergravity-matter systems, the traditional approach can be argued to become impractical and cumbersome (as regards the component tensor calculus employed in [21, 22], it has never been extended, to the best of our knowledge, to the case of the supersymmetric theories we are going to study below, therefore the superspace approach is the only formalism at our disposal). There exist two alternatives [23, 24] to the traditional approach of component reduction [19] that were originally developed for the systems scrutinized in [21, 22] or slightly more generals ones, but remain equally powerful in a more general setting. We prefer to follow the Kugo-Uehara approach [23] that conceptually originates in [25] and is quite natural in the framework of the Siegel-Gates formulation of superfield supergravity [26]. The idea is to follow the pattern of the Weyl invariant extension of Einstein gravity,

$$S[g] = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\,R \quad\longrightarrow\quad S[g,\varphi] = \frac{3}{\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\left\{g^{mn}\,\partial_m\varphi\partial_n\varphi + \frac{1}{6}R\,\varphi^2\right\},$$

and extend any supergravity-matter system to a super-Weyl invariant system (in the Howe-Tucker sense [27]) by introducing a compensating covariantly chiral scalar superfield Σ (in addition to the supergravity chiral compensator [26]). When reducing to components, canonically normalized component actions are obtained simply by imposing a suitable super-Weyl gauge condition to effectively eliminate Σ.

In the present paper, we implement the component reduction scheme of [23] to derive the bosonic action of self-dual supersymmetric nonlinear electrodynamics coupled to the dilaton-axion chiral multiplet and a Kähler sigma-model. As concerns the fermionic sector, the situation is highly nontrivial even in the globally supersymmetric case. The point is that the fermionic action contains higher derivative terms that seem to be removable in the presence of supergravity fields. In the globally supersymmetric case, we explicitly construct a nonlinear field redefinition which eliminates all the higher derivative terms and brings the fermionic action to a one-parameter deformation of the Akulov-Volkov action for the Goldstino [28, 29]. The Akulov-Volkov action emerges, in particular, in the case of the $\mathcal{N}=1$ supersymmetric Born-Infeld action.

This paper is organized as follows. In section 2 we review, following [20], the procedure of reducing locally supersymmetric actions from superfields to components. In section 3 we then spell out the Kugo-Uehara scheme [23] on the example of a nonlinear Kähler sigma-model coupled to supergravity. Section 4 is devoted to the derivation of the bosonic action of self-dual supersymmetric nonlinear electrodynamics coupled to the dilaton-axion chiral multiplet and a Kähler sigma-model. Different aspects of the fermionic dynamics in the globally supersymmetric case are analyzed in sections 5 and 6. A discussion of the results obtained and future perspectives is given in section 7. Some nuances of the Akulov-Volkov (AV) action are presented in appendix A. In particular, we demonstrate that all the terms of eighth order in the AV action completely cancel. Finally, appendix B is devoted to an alternative realization of old minimal supergravity that has not been discussed in textbooks [18 – 20].

2. From superfield supergravity to components

Here we recall salient points of the old minimal [15, 16] and the new minimal [17] formulations of $\mathcal{N}=1$ supergravity (see [18 – 20] for more details), and also review, following [20], the procedure of reducing locally supersymmetric actions from superfields to components.

2.1 Old minimal supergravity

We follow the notation[1] and $\mathcal{N}=1$ supergravity conventions of [20]. Unless otherwise stated we work with the old minimal formulation of $\mathcal{N}=1$ supergravity. The superspace

[1]In particular, $z^M = (x^m, \theta^\mu, \bar\theta_{\dot\mu})$ are the coordinates of $\mathcal{N}=1$ curved superspace, $\mathrm{d}^8z = \mathrm{d}^4x\,\mathrm{d}^2\theta\,\mathrm{d}^2\bar\theta$ is the full flat superspace measure, and $\mathrm{d}^6z = \mathrm{d}^4x\,\mathrm{d}^2\theta$ is the measure in the chiral subspace.

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geometry is described by covariant derivatives [15]

$$\begin{aligned}\mathcal{D}_A &= (\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot\alpha}) = E_A + \Omega_A\,,\\ E_A &= E_A{}^M(z)\partial_M\,, \qquad \Omega_A = \frac{1}{2}\,\Omega_A{}^{bc}(z)M_{bc} = \Omega_A{}^{\beta\gamma}(z)M_{\beta\gamma} + \Omega_A{}^{\dot\beta\dot\gamma}(z)\bar{M}_{\dot\beta\dot\gamma}\,,\end{aligned} \tag{2.1}$$

with E_A^M the vielbein, Ω_A the Lorentz connection and $M_{bc} \Leftrightarrow (M_{\beta\gamma}, \bar{M}_{\dot\beta\dot\gamma})$ the Lorentz generators. The covariant derivatives obey the following algebra [15]:

$$\begin{aligned}\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\} &= -2\mathrm{i}\mathcal{D}_{\alpha\dot\alpha}\,,\\ \{\mathcal{D}_\alpha, \mathcal{D}_\beta\} &= -4\bar{R}M_{\alpha\beta}\,, \qquad \{\bar{\mathcal{D}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\} = -4R\bar{M}_{\dot\alpha\dot\beta}\,,\\ \Big[\bar{\mathcal{D}}_{\dot\alpha}, \mathcal{D}_{\beta\dot\beta}\Big] &= -\mathrm{i}\varepsilon_{\dot\alpha\dot\beta}\Big(R\,\mathcal{D}_\beta + G_\beta{}^{\dot\gamma}\bar{\mathcal{D}}_{\dot\gamma} - (\bar{\mathcal{D}}^{\dot\gamma}G_\beta{}^{\dot\delta})\bar{M}_{\dot\gamma\dot\delta} + 2W_\beta{}^{\gamma\delta}M_{\gamma\delta}\Big) - \mathrm{i}(\mathcal{D}_\beta R)\bar{M}_{\dot\alpha\dot\beta}\,,\\ \Big[\mathcal{D}_\alpha, \mathcal{D}_{\beta\dot\beta}\Big] &= \mathrm{i}\varepsilon_{\alpha\beta}\Big(\bar{R}\,\bar{\mathcal{D}}_{\dot\beta} + G^\gamma{}_{\dot\beta}\mathcal{D}_\gamma - (\mathcal{D}^\gamma G^\delta{}_{\dot\beta})M_{\gamma\delta} + 2\bar{W}_{\dot\beta}{}^{\dot\gamma\dot\delta}\bar{M}_{\dot\gamma\dot\delta}\Big) + \mathrm{i}(\bar{\mathcal{D}}_{\dot\beta}\bar{R})M_{\alpha\beta}\,,\end{aligned} \tag{2.2}$$

where the tensors R, $G_a = \bar{G}_a$ and $W_{\alpha\beta\gamma} = W_{(\alpha\beta\gamma)}$ satisfy the Bianchi identities

$$\bar{\mathcal{D}}_{\dot\alpha}R = \bar{\mathcal{D}}_{\dot\alpha}W_{\alpha\beta\gamma} = 0\,, \qquad \bar{\mathcal{D}}^{\dot\gamma}G_{\alpha\dot\gamma} = \mathcal{D}_\alpha R\,, \qquad \mathcal{D}^\gamma W_{\alpha\beta\gamma} = \mathrm{i}\,\mathcal{D}_{(\alpha}{}^{\dot\gamma}G_{\beta)\dot\gamma}\,. \tag{2.3}$$

Modulo purely gauge degrees of freedom, all geometric objects — the vielbein and the connection — can be expressed [26] in terms of three unconstrained superfields (known as the prepotentials of old minimal supergravity): gravitational superfield $H^m = \bar{H}^m$, chiral compensator φ ($\bar{E}_{\dot\alpha}\varphi = 0$) and its conjugate $\bar\varphi$. The old minimal supergravity action [15] is

$$S_{\mathrm{SG,old}} = -3\int \mathrm{d}^8z\,E^{-1}\,, \qquad E = \mathrm{Ber}(E_A{}^M)\,, \tag{2.4}$$

with the gravitational coupling constant being set equal to one.

2.2 New minimal supergravity

We will also deal with the new minimal formulation of supergravity [17]. This can be treated (see [20] for a review) as a super-Weyl invariant dynamical system that couples old minimal supergravity to the improved tensor multiplet [30] described by a *real* covariantly linear scalar superfield $\mathbb{L}$ [31],

$$(\bar{\mathcal{D}}^2 - 4R)\,\mathbb{L} = (\mathcal{D}^2 - 4\bar{R})\,\mathbb{L} = 0\,. \tag{2.5}$$

Super-Weyl transformations, originally introduced in [27], are simply local rescalings of the chiral compensator in old minimal supergravity [26] (see also [18, 20]),

$$\varphi \;\to\; \mathrm{e}^\sigma\,\varphi\,, \tag{2.6}$$

with $\sigma(z)$ an arbitrary covariantly chiral scalar parameter, $\bar{\mathcal{D}}_{\dot\alpha}\sigma = 0$. In terms of the covariant derivatives, the transformation[2] is

$$\mathcal{D}_\alpha \;\to\; \mathrm{e}^{\sigma/2-\bar\sigma}\Big(\mathcal{D}_\alpha - (\mathcal{D}^\beta\sigma)\,M_{\alpha\beta}\Big)\,, \qquad \bar{\mathcal{D}}_{\dot\alpha} \;\to\; \mathrm{e}^{\bar\sigma/2-\sigma}\Big(\bar{\mathcal{D}}_{\dot\alpha} - (\bar{\mathcal{D}}^{\dot\beta}\bar\sigma)\bar{M}_{\dot\beta\dot\alpha}\Big)\,. \tag{2.7}$$

[2]Under (2.7), the full superspace measure changes as $\mathrm{d}^8z\,E^{-1} \to \mathrm{d}^8z\,E^{-1}\,\exp(\sigma+\bar\sigma)$, while the *chiral superspace measure* transforms as $\mathrm{d}^8z\,E^{-1}/R \to \mathrm{d}^8z\,(E^{-1}/R)\,\exp(3\sigma)$.

Since

$$(\mathcal{D}^2 - 4\bar{R}) \;\to\; \mathrm{e}^{-2\bar{\sigma}}\,(\mathcal{D}^2 - 4\bar{R})\,\mathrm{e}^{\sigma} \tag{2.8}$$

when acting on a scalar superfield, it is clear that the super-Weyl transformation law of $\mathbb{L}$ is uniquely fixed to be

$$\mathbb{L} \;\to\; \mathrm{e}^{-\sigma-\bar{\sigma}}\,\mathbb{L}\,. \tag{2.9}$$

In superspace, the new minimal supergravity action is [32]

$$S_{\mathrm{SG,new}} = 3\int \mathrm{d}^8z\,E^{-1}\,\mathbb{L}\,\mathrm{ln}\mathbb{L}\,. \tag{2.10}$$

Any system of matter superfields Ψ coupled to new minimal supergravity can be treated as a super-Weyl invariant coupling of old minimal supergravity to the matter superfields Ψ and $\mathbb{L}$ [33].

Old minimal and new minimal supergravities are dual to each other. One can show this by considering the “first-order” action [32]

$$S = 3\int \mathrm{d}^8z\,E^{-1}\,(U\,\mathbb{L} - \mathrm{e}^{U})\,, \tag{2.11}$$

where $U(z)$ is an arbitrary real scalar superfield. For (2.11) to be super-Weyl invariant, U must transform under super-Weyl transformations as

$$U \;\to\; U - \sigma - \bar{\sigma}\,. \tag{2.12}$$

Solving for the equation of motion of U, we regain the new minimal supergravity action (2.10). On the other hand, the solution to the equation of motion for $\mathbb{L}$ requires that U be the sum of a covariantly chiral scalar superfield and its conjugate,

$$U = \mathrm{ln}\Sigma + \mathrm{ln}\bar{\Sigma}\,, \qquad \bar{\mathcal{D}}_{\dot{\alpha}}\Sigma = 0\,. \tag{2.13}$$

The action (2.11) then becomes

$$\tilde{S}_{\mathrm{SG,old}} = -3\int \mathrm{d}^8z\,E^{-1}\,\bar{\Sigma}\,\Sigma\,, \tag{2.14}$$

where Σ has, in accordance with (2.12), the following super-Weyl transformation

$$\Sigma \;\to\; \mathrm{e}^{-\sigma}\,\Sigma\,. \tag{2.15}$$

An alternative realization of this dynamical system is given in appendix B.

The action (2.14) is the super-Weyl invariant extension of the old minimal supergravity action (2.4). The latter may be recovered by using the super-Weyl gauge freedom to impose the gauge condition $\Sigma = 1$. In what follows, we prefer to use (2.14). Any dynamical system of matter superfields Ψ coupled to old minimal supergravity can be promoted to a a super-Weyl invariant system [25, 23] if the chiral compensator φ [26] is replaced by the super-Weyl invariant combination

$$\varphi \;\to\; \varphi\,\Sigma\,. \tag{2.16}$$

This is equivalent to applying the super-Weyl transformation (2.7) with $\sigma = \Sigma$ (which may be accompanied by a super-Weyl transformation of the matter superfields).

2.3 Components in old minimal supergravity

The old minimal supergravity multiplet $\{e_a{}^m, \Psi_a{}^\beta, \bar{\Psi}_{a\dot{\beta}}, A_a, B, \bar{B}\}$ comprises the (inverse) vierbein $e_a{}^m$, the gravitino $\mathbf{\Psi}_a = (\Psi_a{}^\beta, \bar{\Psi}_{a\dot{\beta}})$, and the auxiliary fields[3] A_a, B and $\bar{B}$ [16]. Within the framework of superfield supergravity [15], these component fields naturally appear in a Wess-Zumino gauge [34] (see [18 – 20] for reviews). Here we use the Wess-Zumino gauge chosen in [20].

We define superfields' component fields by space projection and covariant differentiation. For a superfield $V(z)$, the former is the zeroth order term in the power series expansion in θ and $\bar{\theta}$

$$V| = V(x, \theta = 0, \bar{\theta} = 0)\,. \tag{2.17}$$

The space projection of the vector covariant derivatives are

$$\mathcal{D}_a| = \nabla_a - \frac{1}{3}\varepsilon_{abcd}\, A^d M^{bc} + \frac{1}{2}\Psi_a{}^\beta\, \mathcal{D}_\beta| + \frac{1}{2}\bar{\Psi}_{a\dot{\beta}}\, \bar{\mathcal{D}}^{\dot{\beta}}|\,, \tag{2.18}$$

where we have introduced the spacetime covariant derivatives, $\nabla_a = e_a + \frac{1}{2}\omega_{abc}M^{bc}$, with $\omega_{abc} = \omega_{abc}(e, \Psi)$ the connection and $e_a = e_a{}^m\partial_m$. The explicit expressions for the projections $\mathcal{D}_\alpha|$ and $\bar{\mathcal{D}}^{\dot{\alpha}}|$ can be found in [20]. The spacetime covariant derivatives obey the following algebra

$$[\nabla_a, \nabla_b] = \mathcal{T}_{ab}{}^c\, \nabla_c + \frac{1}{2}\,\mathcal{R}_{abcd}M^{cd}\,, \tag{2.19}$$

where $\mathcal{R}_{abcd}$ is the curvature tensor and $\mathcal{T}_{abc}$ is the torsion. The torsion is related to the gravitino by

$$\mathcal{T}_{abc} = -\frac{\mathrm{i}}{2}(\Psi_a\sigma_c\bar{\Psi}_b - \Psi_b\sigma_c\bar{\Psi}_a)\,. \tag{2.20}$$

Additionally we can write the connection in terms of the supergravity fields as

$$\omega_{abc} = \omega_{abc}(e) - \frac{1}{2}(\mathcal{T}_{bca} + \mathcal{T}_{acb} - \mathcal{T}_{abc})\,, \qquad \omega_{abc}(e) = \frac{1}{2}(\mathcal{C}_{bca} + \mathcal{C}_{acb} - \mathcal{C}_{abc})\,, \tag{2.21}$$

where $\mathcal{C}_{abc}$ are the anholonomy coefficients,

$$[e_a, e_b] = \mathcal{C}_{ab}{}^c e_c\,, \qquad \mathcal{C}_{ab}{}^c = ((e_a e_b{}^m) - (e_b e_a{}^m))\, e_m{}^c\,. \tag{2.22}$$

The supergravity auxiliary fields occur as follows [34]

$$R| = \frac{1}{3}B\,, \qquad G_a| = \frac{4}{3}A_a\,. \tag{2.23}$$

One also has

$$\begin{aligned}
\mathcal{D}_\alpha R| &= -\frac{2}{3}(\sigma^{bc}\Psi_{bc})_\alpha - \frac{2\mathrm{i}}{3}A^b\Psi_{b\alpha} + \frac{\mathrm{i}}{3}\bar{B}(\sigma^b\bar{\Psi}_b)_\alpha\,,\\
\bar{\mathcal{D}}_{(\dot{\alpha}}G^\beta{}_{\dot{\beta})}| &= -2\Psi_{\dot{\alpha}\dot{\beta},}{}^\beta + \frac{\mathrm{i}}{3}\bar{B}\bar{\Psi}^\beta{}_{(\dot{\alpha},\dot{\beta})} - 2\mathrm{i}(\tilde{\sigma}^{ab})_{\dot{\alpha}\dot{\beta}}\Psi_a{}^\beta A_b + \frac{2\mathrm{i}}{3}\Psi_{\alpha(\dot{\alpha},}{}^\alpha A_{\dot{\beta})}{}^\beta\,,\\
W_{\alpha\beta\gamma}| &= \Psi_{(\alpha\beta,\gamma)} - \mathrm{i}(\sigma_{ab})_{(\alpha\beta}\Psi^a{}_{\gamma)}A^b\,,
\end{aligned} \tag{2.24}$$

[3] These auxiliary fields are denoted as $\mathbf{A}_a$, $\mathbf{B}$ and $\bar{\mathbf{B}}$ in [20].

and

$$\bar{\mathcal{D}}^2\bar{R}| = \frac{2}{3}\left(\mathcal{R} + \frac{\mathrm{i}}{2}\varepsilon^{abcd}\,\mathcal{R}_{abcd}\right) + \frac{16}{9}A^aA_a + \frac{4}{9}\epsilon^{abcd}\mathcal{T}_{abc}A_d - \frac{8\mathrm{i}}{3}(\nabla_aA^a) + \frac{8\mathrm{i}}{9}\mathcal{T}_{ab}{}^bA^a + \\ + \frac{8}{9}B\bar{B} + \frac{4}{9}B(\Psi_a\sigma^{ab}\Psi_b) + \mathrm{i}\bar{\mathcal{D}}_{\dot\alpha}\bar{R}|(\tilde\sigma^a\Psi_a)^{\dot\alpha} + \frac{2\mathrm{i}}{3}\Psi^{\alpha\dot\alpha,\beta}\mathcal{D}_{(\alpha}G_{\beta)\dot\alpha}|\,, \tag{2.25}$$

where

$$\begin{aligned}\Psi_{ab}{}^\gamma &= \nabla_a\Psi_b{}^\gamma - \nabla_b\Psi_a{}^\gamma - \mathcal{T}_{ab}{}^c\Psi_c{}^\gamma\,,\\ \Psi_{\alpha\beta,}{}^\gamma &= \frac{1}{2}(\sigma^{ab})_{\alpha\beta}\Psi_{ab}{}^\gamma\,, \qquad \Psi_{\dot\alpha\dot\beta,}{}^\gamma = -\frac{1}{2}(\tilde\sigma^{ab})_{\dot\alpha\dot\beta}\Psi_{ab}{}^\gamma\,.\end{aligned} \tag{2.26}$$

With these objects and the covariant derivative algebra, (2.2) the method to obtain the component action is as follows.

Since

$$\int \mathrm{d}^8z\,E^{-1}\,\mathcal{L} = -\frac{1}{4}\int \mathrm{d}^8z\,\frac{E^{-1}}{R}\,(\bar{\mathcal{D}}^2 - 4R)\mathcal{L}\,, \tag{2.27}$$

modulo a total derivative, it is sufficient to work with chiral actions involving a chiral scalar lagrangian $\mathcal{L}_\mathrm{c}$, $\bar{\mathcal{D}}_{\dot\alpha}\mathcal{L}_\mathrm{c} = 0$. Such a chiral action generates the following component action [20]

$$\begin{aligned}\int \mathrm{d}^8z\,\frac{E^{-1}}{R}\mathcal{L}_\mathrm{c} &= \int \mathrm{d}^4x\,e^{-1}\Big\{-\frac{1}{4}\mathcal{D}^2\mathcal{L}_\mathrm{c}| - \frac{\mathrm{i}}{2}(\bar\Psi^b\tilde\sigma_b)^\alpha\,\mathcal{D}_\alpha\mathcal{L}_\mathrm{c}| + (B + \bar\Psi^a\tilde\sigma_{ab}\bar\Psi^b)\,\mathcal{L}_\mathrm{c}|\Big\}\,,\\ e &= \det(e_a{}^m)\,.\end{aligned} \tag{2.28}$$

The component action for old minimal supergravity, eq. (2.4), is [16]

$$S_{\mathrm{SG,old}} = \int \mathrm{d}^4x\,e^{-1}\Big\{\frac{1}{2}\mathcal{R} + \frac{4}{3}A^aA_a - \frac{1}{3}\bar{B}B + \frac{1}{4}\varepsilon^{abcd}(\bar\Psi_a\tilde\sigma_b\Psi_{cd} - \Psi_a\sigma_b\bar\Psi_{cd})\Big\}\,, \tag{2.29}$$

see [19, 20] for reviews.

3. Kähler sigma-models in supergravity

To illustrate the Kugo-Uehara approach to component reduction [23], we consider a non-linear Kähler sigma-model coupled to supergravity.

3.1 Superfield formulations

Kähler sigma-models are most easily described within the framework of new minimal supergravity [33] (see [20] for a review). Given a Kähler manifold parametrized by n complex coordinates ϕ^i and their conjugates $\bar\phi^{\underline{i}}$, with $K(\phi,\bar\phi)$ the Kähler potential, the corresponding supergravity-matter action is

$$S = 3\int \mathrm{d}^8z\,E^{-1}\,\mathbb{L}\,\ln\mathbb{L} + \int \mathrm{d}^8z\,E^{-1}\,\mathbb{L}\,K(\phi,\bar\phi)\,. \tag{3.1}$$

The dynamical variables ϕ are covariantly chiral scalar superfields, $\bar{\mathcal{D}}_{\dot{\alpha}}\phi = 0$, being inert with respect to the super-Weyl transformations. The action is obviously super-Weyl invariant. Due to (2.27), it is also invariant under the Kähler transformations

$$K(\phi, \bar{\phi}) \rightarrow K(\phi, \bar{\phi}) + \lambda(\phi) + \bar{\lambda}(\bar{\phi}), \tag{3.2}$$

with $\lambda(\phi)$ an arbitrary holomorphic function.

To reformulate the dynamics within the framework of old minimal supergravity, let us introduce the auxiliary action

$$S = 3 \int \mathrm{d}^8 z\, E^{-1} \left(U \,\mathbb{L} - \Upsilon \right), \tag{3.3}$$

where

$$\Upsilon = \exp\left(U - \frac{1}{3} K(\phi, \bar{\phi}) \right), \tag{3.4}$$

and U is an arbitrary real scalar superfield. For the action (3.3) to be super-Weyl invariant U must transform by the law (2.12) under transformations (2.7). To preserve Kähler invariance (3.2), the Kähler transformation of U should be

$$U \;\rightarrow\; U + \frac{1}{3}\left(\lambda(\phi) + \bar{\lambda}(\bar{\phi}) \right). \tag{3.5}$$

Solving the equation of motion for U, we regain the supergravity-matter action (3.1). On the other hand, the solution to the equation of motion for $\mathbb{L}$ requires that U be the sum of a covariantly chiral scalar superfield and its conjugate, as given by (2.13). The Kähler sigma-model then reads

$$S_{\mathrm{Kahler}} = -3 \int \mathrm{d}^8 z\, E^{-1}\, \bar{\Sigma}\, \Sigma \exp\left(-\frac{1}{3} K(\phi, \bar{\phi}) \right) = -3 \int \mathrm{d}^8 z\, E^{-1}\, \tilde{\Upsilon}, \tag{3.6}$$

where Υ is as defined in (3.4) but with U given by the solution (2.13)

$$\tilde{\Upsilon} = \Sigma \bar{\Sigma} \exp\left(-\frac{1}{3} K(\phi, \bar{\phi}) \right). \tag{3.7}$$

Super-Weyl transformations of Σ are given by (2.15), whereas under the Kähler transformations (3.2) we have

$$\Sigma \;\rightarrow\; \mathrm{e}^{\lambda(\phi)/3}\, \Sigma. \tag{3.8}$$

3.2 Component action of Kähler sigma-model

To determine the component structure of (3.6), the approach of [23] requires that a particular super-Weyl gauge choice be made such that the Einstein and Rarita-Schwinger terms in the component action come out in canonical form.

We define[4] the component fields of the chiral scalar superfields ϕ^i by

$$\phi^i| = Y^i, \qquad \mathcal{D}_\alpha \phi^i| = \chi^i_\alpha, \qquad -\frac{1}{4}\mathcal{D}^2 \phi^i| = F^i + \frac{1}{4}\Gamma^i{}_{jk}\, \chi^j \chi^k, \tag{3.9}$$

[4]With such a definition, both χ^i and F^i transform as tangent vectors under arbitrary holomorphic reparametrizations, $Y^i \rightarrow f^i(Y)$, of the Kähler manifold with Kähler potential $K(Y, \bar{Y})$.

where we have introduced the Christoffel symbols $\Gamma^i{}_{jk}$ of the Kähler manifold defined by Kähler potential $K(Y, \bar{Y})$. The metric of the Kähler manifold is

$$g_{i\underline{i}} = g_{\underline{i}i} = \frac{\partial^2 K(Y,\bar{Y})}{\partial Y^i \partial \bar{Y}^{\underline{i}}} \equiv K_{i\underline{i}}\,, \tag{3.10}$$

where the subscript i ($\underline{i}$) on K denotes differentiation with respect to Y^i ($\bar{Y}^{\underline{i}}$). Similarly, we can write expressions for the Christoffel symbols and the curvature on the Kähler manifold

$$\begin{aligned} \Gamma^i{}_{jk} &= g^{i\underline{i}} K_{jk\underline{i}}\,, \qquad \Gamma^{\underline{i}}{}_{\underline{jk}} = g^{\underline{i}i} K_{i\underline{jk}}\,, \\ \mathrm{R}_{ij\underline{ij}} &= K_{ij\underline{ij}} - g^{k\underline{k}} K_{ij\underline{k}} K_{k\underline{ij}}\,, \end{aligned} \tag{3.11}$$

where the matrix elements $g^{i\underline{i}} = g^{\underline{i}i}$ correspond to the inverse Kähler metric, $g^{i\underline{j}}\, g_{\underline{j}k} = \delta^i{}_k$.

Applying the reduction formula (2.28) to the Kähler sigma model (3.6) we obtain

$$\begin{aligned} S_{\text{Kahler}} = -3 \int \mathrm{d}^4x\, e^{-1} \Big\{ &\Big(-\frac{1}{4}\mathcal{D}^2 R| - \frac{\mathrm{i}}{2}(\bar{\Psi}^a \tilde{\sigma}_a)^\alpha \mathcal{D}_\alpha R| + (B + \bar{\Psi}^a \tilde{\sigma}_{ab} \bar{\Psi}^b) R| \Big) \tilde{\Upsilon}| - \\ &- \frac{1}{2}\mathcal{D}^\alpha R| \mathcal{D}_\alpha \tilde{\Upsilon}| - \frac{1}{4} R| \mathcal{D}^2 \tilde{\Upsilon}| - \frac{\mathrm{i}}{2}(\bar{\Psi}^a \tilde{\sigma}_a)^\alpha R| \mathcal{D}_\alpha \tilde{\Upsilon}| - \\ &- \frac{1}{16}\mathcal{D}^2 \bar{\mathcal{D}}^2 \tilde{\Upsilon}| + \frac{\mathrm{i}}{8}(\bar{\Psi}^a \tilde{\sigma}_a)^\alpha \mathcal{D}_\alpha \bar{\mathcal{D}}^2 \tilde{\Upsilon}| - \frac{1}{4}(B + \bar{\Psi}^a \tilde{\sigma}_{ab} \bar{\Psi}^b) \bar{\mathcal{D}}^2 \tilde{\Upsilon}| \Big\}\,. \end{aligned} \tag{3.12}$$

The first line of (3.12) reduces to the supergravity action (2.29) if we make a super-Weyl gauge choice such that $\tilde{\Upsilon}| = 1$. This can be done by setting

$$\Sigma| = \mathrm{e}^{K(Y,\bar{Y})/6}\,. \tag{3.13}$$

We have now eliminated the need to perform a Weyl rescaling on the component action. Further specification of the components of Σ can be used to remove the need for the chiral rotation (and gravitino shift). We accomplish this with the following choices

$$\begin{aligned} \mathcal{D}_\alpha \Sigma| &= \frac{1}{3}\, \chi^i_\alpha K_i\, \mathrm{e}^{K(Y,\bar{Y})/6}\,, \\ -\frac{1}{4}\mathcal{D}^2 \Sigma| &= \Big(\frac{1}{3} F^i K_i - \frac{1}{12}\, \chi^i \chi^j (K_{ij} - \Gamma^k{}_{ij} K_k + \frac{1}{3} K_i K_j) \Big)\, \mathrm{e}^{K(Y,\bar{Y})/6}\,. \end{aligned} \tag{3.14}$$

Such a choice implies that $\mathcal{D}_\alpha \tilde{\Upsilon}| = \mathcal{D}^2 \tilde{\Upsilon}| = 0$, and thus the second line of (3.12) vanishes. The action reduces to

$$S_{\text{Kahler}} = S_{\text{SG,old}} - 3 \int \mathrm{d}^4x\, e^{-1} \Big\{ -\frac{1}{16}\mathcal{D}^2 \bar{\mathcal{D}}^2 \tilde{\Upsilon}| + \frac{\mathrm{i}}{8}(\bar{\Psi}^a \tilde{\sigma}_a)^\alpha \mathcal{D}_\alpha \bar{\mathcal{D}}^2 \tilde{\Upsilon}| \Big\}\,. \tag{3.15}$$

We are now in a position to write down the component action for supergravity coupled to a Kähler sigma model. This result, which is in agreement with the original considerations [21, 22] (see [19] for a review), is

$$S_{\text{Kahler}} = \int \mathrm{d}^4x\, e^{-1} \Big\{ \frac{1}{2}\mathcal{R} + \frac{4}{3}\mathbb{A}^a \mathbb{A}_a - \frac{1}{3}\bar{B}B + \frac{1}{4}\varepsilon^{abcd}(\bar{\Psi}_a \tilde{\sigma}_b \hat{\Psi}_{cd} - \Psi_a \sigma_b \hat{\bar{\Psi}}_{cd}) -$$

$$
\begin{aligned}
& - g_{i\underline{i}}\Big(\nabla^a Y^i\, \nabla_a \bar{Y}^{\underline{i}} + \frac{\mathrm{i}}{4}(\chi^i \sigma^a \overset{\leftrightarrow}{\hat{\nabla}}_a\, \bar{\chi}^{\underline{i}}) - F^i \bar{F}^{\underline{i}} - \\
&\qquad - \frac{1}{2}(\Psi_a \sigma^b \tilde{\sigma}^a \chi^i)(\nabla_b \bar{Y}^{\underline{i}}) - \frac{1}{2}(\bar{\Psi}_a \tilde{\sigma}^b \sigma^a \bar{\chi}^{\underline{i}})(\nabla_b Y^i) - \qquad (3.16)\\
&\qquad - \frac{1}{8}(\Psi^a \sigma_b \bar{\Psi}_a)(\chi^i \sigma^b \bar{\chi}^{\underline{i}}) - \frac{\mathrm{i}}{8}\varepsilon^{abcd}(\Psi_a \sigma_b \bar{\Psi}_c)(\chi^i \sigma_d \bar{\chi}^{\underline{i}})\Big) + \\
& + \frac{1}{16}\chi^i \chi^j \bar{\chi}^{\underline{i}} \bar{\chi}^{\underline{j}}\Big(\mathrm{R}_{ij\underline{i}\underline{j}} - \frac{1}{2} g_{i\underline{i}} g_{j\underline{j}}\Big)\Big\}\,,
\end{aligned}
$$

where

$$
\begin{aligned}
\hat{\Psi}_{ab}{}^{\gamma} &= \hat{\nabla}_a \Psi_b{}^{\gamma} - \hat{\nabla}_b \Psi_a{}^{\gamma} - \mathcal{T}_{ab}{}^{c} \Psi_c{}^{\gamma}\,,\\
\hat{\nabla}_a \Psi_b{}^{\gamma} &= \nabla_a \Psi_b{}^{\gamma} + \frac{1}{4}\left(K_i\, \nabla_a Y^i - K_{\underline{i}}\, \nabla_a \bar{Y}^{\underline{i}}\right) \Psi_b{}^{\gamma}\,,\\
\hat{\nabla}_a \chi^i_{\gamma} &= \nabla_a \chi^i_{\gamma} - \frac{1}{4}\left(K_j\, \nabla_a Y^j - K_{\underline{j}}\, \nabla_a \bar{Y}^{\underline{j}}\right) \chi^i_{\gamma} + \Gamma^i{}_{jk}(\nabla_a Y^j) \chi^k_{\gamma}\,.
\end{aligned}
\qquad (3.17)
$$

In order to to diagonalize in the auxiliary field sector we have made the redefinition

$$
A_a = \mathbb{A}_a - \frac{\mathrm{i}}{4}\left(K_i\, \nabla_a Y^i - K_{\underline{i}}\, \nabla_a \bar{Y}^{\underline{i}}\right) - \frac{1}{16}\, g_{i\underline{i}} (\chi^i \sigma_a \bar{\chi}^{\underline{i}})\,, \qquad (3.18)
$$

so that the auxiliary fields $\mathbb{A}_a, B$ and F^i vanish on the mass shell.

4. Self-dual electrodynamics in supergravity

We are finally prepared to study supersymmetric nonlinear electrodynamics.

4.1 Family of self-dual models

In [14] we constructed a family of self-dual models for the abelian vector multiplet in curved superspace with actions of the general form

$$
S[W, \bar{W}] = \frac{1}{4}\int \mathrm{d}^8 z\, \frac{E^{-1}}{R}\, W^2 + \frac{1}{4}\int \mathrm{d}^8 z\, \frac{E^{-1}}{\bar{R}}\, \bar{W}^2 + \frac{1}{4}\int \mathrm{d}^8 z\, E^{-1}\, W^2\, \bar{W}^2\, \Lambda(\omega, \bar{\omega})\,, \quad (4.1)
$$

where $\Lambda(\omega, \bar{\omega})$ is a real analytic function of the complex variable

$$
\omega \equiv \frac{1}{8}(\mathcal{D}^2 - 4\bar{R})\, W^2\,. \qquad (4.2)
$$

Here $\bar{W}_{\dot{\alpha}}$ and W_{α} are covariantly (anti) chiral superfield strengths,

$$
W_{\alpha} = -\frac{1}{4}\,(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_{\alpha}\, V\,, \qquad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}\,(\mathcal{D}^2 - 4\bar{R})\bar{\mathcal{D}}_{\dot{\alpha}}\, V\,, \qquad (4.3)
$$

defined in terms of a real unconstrained prepotential V. The theory (4.1) is self-dual if the interaction $\Lambda(\omega, \bar{\omega})$ satisfies the following differential equation

$$
\mathrm{Im}\left\{\Gamma - \bar{\omega}\,\Gamma^2\right\} = 0\,, \qquad \Gamma = \frac{\partial(\omega\, \Lambda)}{\partial \omega}\,. \qquad (4.4)
$$

The self-dual dynamical systems described are a curved-superspace generalization of the globally supersymmetric systems introduced in [8, 9].

To obtain a super-Weyl invariant extension of (4.1), we first note that V is inert under the super-Weyl transformation (2.7), and therefore the chiral strength W_α transforms as follows

$$W_\alpha \;\to\; \mathrm{e}^{-3\sigma/2}\, W_\alpha\,. \tag{4.5}$$

Now, implementing the substitution (2.16) in $S[W,\bar W]$, with the aid of (2.8), we then obtain the super-Weyl invariant action

$$S[W,\bar W,\Sigma,\bar\Sigma] = S[W,\bar W,\Sigma\bar\Sigma]\,, \tag{4.6}$$

where

$$S[W,\bar W,\Upsilon] = \frac{1}{4}\int \mathrm{d}^8 z\, \frac{E^{-1}}{R}\, W^2 + \frac{1}{4}\int \mathrm{d}^8 z\, \frac{E^{-1}}{\bar R}\, \bar W^2 + \frac{1}{4}\int \mathrm{d}^8 z\, E^{-1}\, \frac{W^2\,\bar W^2}{\Upsilon^2}\,\Lambda\Big(\frac{\omega}{\Upsilon^2},\frac{\bar\omega}{\Upsilon^2}\Big)\,. \tag{4.7}$$

4.2 Coupling to Kähler sigma-models

In the presence of a nonlinear Kähler sigma-model, the simplest approach to obtain a super-Weyl and Kähler invariant formulation of dynamics is to proceed within the framework of new minimal supergravity. Consider the supergravity-matter system described by the action [14]

$$S[W,\bar W,\phi,\bar\phi,\mathbb{L}] = 3\int \mathrm{d}^8 z\, E^{-1}\,\mathbb{L}\ln\mathbb{L} + \int \mathrm{d}^8 z\, E^{-1}\,\mathbb{L}\, K(\phi,\bar\phi) + S[W,\bar W,\mathbb{L}]\,, \tag{4.8}$$

where $S[W,\bar W,\mathbb{L}]$ is obtained from (4.7) by replacing $\Upsilon \to \mathbb{L}$. This theory possesses several important symmetries: (i) super-Weyl invariance; (ii) Kähler invariance; (iii) duality invariance.

To uncover the description of this theory in the framework of old minimal supergravity, let us replace the action (4.8) by the following auxiliary action

$$S[W,\bar W,\phi,\bar\phi,\mathbb{L},U] = 3\int \mathrm{d}^8 z\, E^{-1}\,(U\,\mathbb{L} - \Upsilon) + S[W,\bar W,\Upsilon]\,, \tag{4.9}$$

where

$$\Upsilon = \exp\left(U - \frac{1}{3}K(\phi,\bar\phi)\right)\,. \tag{4.10}$$

Here the additional dynamical variable U is an unconstrained real scalar superfield. Varying U brings us back to (4.8). On the other hand, the equation of motion for $\mathbb{L}$ implies that U takes the form (2.13). We thus end up with the action

$$\begin{aligned} S[W,\bar W,\phi,\bar\phi,\Sigma,\bar\Sigma] &= -3\int \mathrm{d}^8 z\, E^{-1}\,\tilde\Upsilon + S[W,\bar W,\tilde\Upsilon]\,,\\ \tilde\Upsilon &= \Sigma\bar\Sigma\exp\Big(-\frac{1}{3}K(\phi,\bar\phi)\Big)\,. \end{aligned} \tag{4.11}$$

4.3 Coupling to the dilaton-axion multiplet

As demonstrated in [14], the supergravity-matter system (4.11) enjoys invariance under electromagnetic duality rotations which do not act on the supergravity prepotentials and sigma-model fields. The duality group can be shown to be U(1). Building on the ideas developed, in particular, in [3, 4, 9], one can enhance the duality group to $\mathrm{SL}(2,\mathbb{R})$ by coupling the vector multiplet in (4.11) to the dilaton-axion multiplet that transforms under duality rotations. The dilaton-axion complex is described by a covariantly chiral scalar superfield, Φ, and takes its values in the Kähler manifold $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$. This program was explicitly realized in [14]. The super-Weyl invariant extension of the action given in [14] is

$$\begin{aligned} S = &-3\int \mathrm{d}^8 z\, E^{-1}\, \tilde{\mathbf{\Upsilon}} + \frac{\mathrm{i}}{4}\int \mathrm{d}^8 z\, \frac{E^{-1}}{R}\, \Phi\, W^2 - \frac{\mathrm{i}}{4}\int \mathrm{d}^8 z\, \frac{E^{-1}}{\bar{R}}\, \bar{\Phi}\, \bar{W}^2 - \\ &-\frac{1}{16}\int \mathrm{d}^8 z\, E^{-1}\, (\Phi-\bar{\Phi})^2\, \frac{W^2\,\bar{W}^2}{\tilde{\mathbf{\Upsilon}}^2}\, \Lambda\Big(\frac{\mathrm{i}}{2}(\Phi-\bar{\Phi})\,\frac{\omega}{\tilde{\mathbf{\Upsilon}}^2}\,,\ \frac{\mathrm{i}}{2}(\Phi-\bar{\Phi})\,\frac{\bar{\omega}}{\tilde{\mathbf{\Upsilon}}^2}\Big)\,, \end{aligned} \tag{4.12}$$

where

$$\tilde{\mathbf{\Upsilon}} = \Sigma\bar{\Sigma}\exp\Big(-\frac{1}{3}\mathcal{K}(\Phi,\bar{\Phi}) - \frac{1}{3}K(\phi,\bar{\phi})\Big)\,. \tag{4.13}$$

Here $\mathcal{K}(\Phi,\bar{\Phi})$ denotes the Kähler potential of the manifold $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$. It has the form

$$\mathcal{K}(\Phi,\bar{\Phi}) = -\ln\frac{\mathrm{i}}{2}(\Phi-\bar{\Phi})\,. \tag{4.14}$$

The action (4.11) follows from (4.12) by setting $\Phi = -\mathrm{i}$.

Now, it is our aim to analyze the component structure of the theory with action (4.12).

4.4 Component reduction

We proceed by introducing the component fields of the vector multiplet

$$W_\alpha| = \psi_\alpha\,, \qquad -\frac{1}{2}\mathcal{D}^\alpha W_\alpha| = D\,, \qquad \mathcal{D}_{(\alpha}W_{\beta)}| = 2\mathrm{i}\hat{F}_{\alpha\beta} = \mathrm{i}(\sigma^{ab})_{\alpha\beta}\hat{F}_{ab}\,, \tag{4.15}$$

where

$$\begin{aligned} \hat{F}_{ab} &= F_{ab} - \frac{1}{2}(\Psi_a\sigma_b\bar{\psi} + \psi\sigma_b\bar{\Psi}_a) + \frac{1}{2}(\Psi_b\sigma_a\bar{\psi} + \psi\sigma_a\bar{\Psi}_b)\,, \\ F_{ab} &= \nabla_a V_b - \nabla_b V_a - \mathcal{T}_{ab}{}^c V_c\,, \end{aligned} \tag{4.16}$$

with $V_a = e_a{}^m(x)\,V_m(x)$ the gauge one-form.

Similarly to our definition (3.9) of the component fields $\{Y^i, \chi^i_\alpha, F^i\}$ of the scalar superfield ϕ^i, we introduce the component fields $\{\mathcal{Y}, \eta_\alpha, \mathcal{F}\}$ of the dilaton-axion multiplet Φ. The dilaton[5] φ and axion a fields are related to the superfield Φ by

$$\Phi| = \mathcal{Y} = a - \mathrm{i}\,\mathrm{e}^{-\varphi}\,. \tag{4.17}$$

[5] We have used the same Greek letter φ to denote the chiral prepotential and the dilaton. Only the latter occurs in the remainder of this paper.

Applying the reduction rule (2.28) to the action (4.12) we obtain

$$
\begin{aligned}
S = S_V - 3\int \mathrm{d}^4x\, e^{-1}\bigg\{ & \Big(-\frac{1}{4}\mathcal{D}^2R| - \frac{\mathrm{i}}{2}(\bar{\Psi}^a\tilde{\sigma}_a)^\alpha\mathcal{D}_\alpha R| + (B + \bar{\Psi}^a\tilde{\sigma}_{ab}\bar{\Psi}^b)R|\Big)\Omega| - \\
& - \frac{1}{2}\mathcal{D}^\alpha R|\mathcal{D}_\alpha\Omega| - \frac{1}{4}R|\mathcal{D}^2\Omega| - \frac{\mathrm{i}}{2}(\bar{\Psi}^a\tilde{\sigma}_a)^\alpha\mathcal{D}_\alpha\Omega| + \\
& + \frac{1}{16}\mathcal{D}^2\bar{\mathcal{D}}^2\Omega| + \frac{\mathrm{i}}{8}(\bar{\Psi}^a\tilde{\sigma}_a)^\alpha\mathcal{D}_\alpha\bar{\mathcal{D}}^2\Omega| - \frac{1}{4}(B + \bar{\Psi}^a\tilde{\sigma}_{ab}\bar{\Psi}^b)\bar{\mathcal{D}}^2\Omega|\bigg\},
\end{aligned} \tag{4.18}
$$

where

$$
\Omega = \tilde{\mathbf{\Upsilon}} + \frac{1}{48}(\Phi - \bar{\Phi})^2\frac{W^2\bar{W}^2}{\tilde{\mathbf{\Upsilon}}^2}\,\Lambda\Big(\frac{\mathrm{i}}{2}(\Phi - \bar{\Phi})\,\frac{\omega}{\tilde{\mathbf{\Upsilon}}^2}\,,\ \frac{\mathrm{i}}{2}(\Phi - \bar{\Phi})\,\frac{\bar{\omega}}{\tilde{\mathbf{\Upsilon}}^2}\Big), \tag{4.19}
$$

and we have separated out the following part of the action:

$$
S_V = \frac{\mathrm{i}}{4}\int \mathrm{d}^8z\,\frac{E^{-1}}{R}\,\Phi\,W^2 - \frac{\mathrm{i}}{4}\int \mathrm{d}^8z\,\frac{E^{-1}}{\bar{R}}\,\bar{\Phi}\,\bar{W}^2\,. \tag{4.20}
$$

Since S_V does not couple to Σ and $\bar{\Sigma}$, its component form is independent of the super-Weyl gauge choice. It is therefore straightforward to evaluate the component structure of this part of the action

$$
\begin{aligned}
S_V = \int \mathrm{d}^4x\, e^{-1}\bigg\{ & -\frac{1}{4}\,\mathrm{e}^{-\varphi}\,F^{ab}F_{ab} + \frac{1}{4}\,a\,F^{ab}\tilde{F}_{ab} - \frac{1}{2}(\psi\sigma^b\bar{\psi})\nabla_b a - \frac{\mathrm{i}}{2}\,\mathrm{e}^{-\varphi}\,(\psi\sigma^a\nabla_a\bar{\psi}) + \\
& + \frac{1}{2}\,\mathrm{e}^{-\varphi}\,F^{ab}(\Psi_a\sigma_b\bar{\psi} + \psi\sigma_b\bar{\Psi}_a) + \frac{\mathrm{i}}{2}\,\mathrm{e}^{-\varphi}\,\tilde{F}^{ab}(\Psi_a\sigma_b\bar{\psi} - \psi\sigma_b\bar{\Psi}_a) + \\
& + \frac{1}{4}\,F^{ab}(\eta\sigma_{ab}\psi + \bar{\eta}\tilde{\sigma}_{ab}\bar{\psi}) + \\
& + \frac{1}{4}\Big(\mathrm{e}^{-\varphi}(\eta^{ac}\eta^{bd} - \eta^{ad}\eta^{bc}) + a\,\epsilon^{abcd}\Big)\,(\Psi_a\sigma_b\bar{\psi})(\psi\sigma_c\bar{\Psi}_d) + \\
& + \frac{1}{16}\,\mathrm{e}^{-\varphi}\Big((3\bar{\Psi}^a\bar{\Psi}_a - 2\bar{\Psi}^a\tilde{\sigma}_{ab}\bar{\Psi}^b)\psi^2 + (3\Psi^a\Psi_a - 2\Psi^a\sigma_{ab}\Psi^b)\bar{\psi}^2\Big) - \\
& - \frac{1}{8}(\Psi_a\sigma_b\bar{\psi})(\eta\sigma^{ab}\psi) - \frac{1}{8}(\bar{\Psi}_a\tilde{\sigma}_b\psi)(\bar{\eta}\tilde{\sigma}^{ab}\bar{\psi}) - \frac{1}{32}\psi^2(\eta\sigma^a\bar{\Psi}_a) + \\
& + \frac{1}{32}\bar{\psi}^2(\Psi_a\sigma^a\bar{\eta}) + \frac{1}{16}\,\mathrm{e}^{\varphi}\,(\eta^2\psi^2 + \bar{\eta}^2\bar{\psi}^2) + \frac{1}{2}(\psi\sigma^a\bar{\psi})\mathcal{T}_{ab}{}^b - \\
& - \frac{\mathrm{i}}{4}(\eta\psi - \bar{\eta}\bar{\psi})D + \frac{1}{2}\,\mathrm{e}^{-\varphi}\,D^2 - \mathrm{e}^{-\varphi}(\psi\sigma^a\bar{\psi})A_a + \frac{\mathrm{i}}{4}(\mathcal{F}\psi^2 - \bar{\mathcal{F}}\bar{\psi}^2)\bigg\},
\end{aligned} \tag{4.21}
$$

where we have used the explicit form of the Kähler potential (4.14). This result is in agreement with [24].

Looking at the first line of (4.18) we notice that if a super-Weyl gauge choice is made such that $\Omega| = 1$ then this will reduce to the supergravity action (2.29), and not require a Weyl rescaling. To achieve this, we make the choice

$$
\Sigma| = \exp\left(\frac{1}{6}K(Y,\bar{Y}) + \frac{1}{6}\mathcal{K}(\mathcal{Y},\bar{\mathcal{Y}}) + \frac{1}{24}\mathrm{e}^{-2\varphi}\psi^2\bar{\psi}^2\Lambda\left(\mathrm{e}^{-\varphi}\omega|\,,\ \mathrm{e}^{-\varphi}\bar{\omega}|\right)\right). \tag{4.22}
$$

A number of options are available for the gauge choice for the other components of Σ. If the following gauge choices are made

$$
\mathcal{D}_\alpha\Sigma| = \frac{1}{3}\Big(\chi^i_\alpha K_i - \frac{\mathrm{i}}{2}\,\mathrm{e}^{\varphi}\eta_\alpha\Big)\,\Sigma|\,,
$$

$$
-\frac{1}{4}\mathcal{D}^2\Sigma| = \frac{1}{3}\Big(F^i K_i - \frac{1}{4}\chi^i\chi^j(K_{ij} - \Gamma^k{}_{ij}K_k + \frac{1}{3}K_iK_j) - \\
- \frac{\mathrm{i}}{2}\,\mathrm{e}^{\varphi}\mathcal{F} - \frac{1}{24}\,\mathrm{e}^{2\varphi}\,\eta^2 + \frac{\mathrm{i}}{12}\,\mathrm{e}^{\varphi}\,\eta\chi^i K_i\Big)\Sigma| \,, \tag{4.23}
$$

then $\mathcal{D}_\alpha\tilde{\boldsymbol{\Upsilon}}| = \mathcal{D}^2\tilde{\boldsymbol{\Upsilon}}| = 0$, and the action (4.18) simplifies greatly.

The complete component action turns out to be extremely complicated as far as the fermionic sector is concerned. The fermionic sector will be studied in the flat-space case in sections 5 and 6. Here we only focus on the bosonic sector.

$$
S_{\text{bosonic}} = \int \mathrm{d}^4x\, e^{-1}\Big\{\frac{1}{2}\mathcal{R} - g_{i\underline{i}}\,\nabla^a Y^i\,\nabla_a\bar{Y}^{\underline{i}} - \frac{1}{4}\Big(\mathrm{e}^{2\varphi}\,(\nabla a)^2 + (\nabla\varphi)^2\Big) - \\
- \frac{1}{4}\,\mathrm{e}^{-\varphi}F^{ab}F_{ab} + \frac{1}{4}\,a\,F^{ab}\tilde{F}_{ab} + \mathrm{e}^{-2\varphi}\,w\bar{w}\,\Lambda\big(\mathrm{e}^{-\varphi}w\,,\,\mathrm{e}^{-\varphi}\bar{w}\big) + \\
+ \frac{4}{3}\mathbb{A}^a\mathbb{A}_a - \frac{1}{3}B\bar{B} + \frac{1}{2}\mathrm{e}^{-\varphi}D^2 + g_{i\underline{i}}\,F^i\bar{F}^{\underline{i}} + \frac{1}{4}\,\mathrm{e}^{2\varphi}\,\mathcal{F}\bar{\mathcal{F}}\Big\}\,, \tag{4.24}
$$

where

$$
w = F^{\alpha\beta}F_{\alpha\beta} - \frac{1}{2}D^2\,, \qquad \bar{w} = \bar{F}^{\dot\alpha\dot\beta}\bar{F}_{\dot\alpha\dot\beta} - \frac{1}{2}D^2\,, \\
A_a = \mathbb{A}_a - \frac{\mathrm{i}}{4}\left(K_i\,\nabla_a Y^i - K_{\underline{i}}\,\nabla_a\bar{Y}^{\underline{i}}\right) - \frac{1}{4}\,\mathrm{e}^{\varphi}\,\nabla_a a\,, \tag{4.25}
$$

and $\mathcal{R}$ and F_{ab} are as defined respectively in (2.19) and (4.16), but with torsion set to zero.

As a special representative in the family of self-dual actions (4.1)–(4.4), we would like to consider the supersymmetric Born-Infeld action. In this case the function $\Lambda(\omega,\bar{\omega})$ takes the form

$$
\Lambda(\omega,\bar{\omega}) = \frac{\kappa^2}{1 + \frac{1}{2}\,A + \sqrt{1 + A + \frac{1}{4}\,B^2}}\,, \\
A = \kappa^2(\omega + \bar{\omega})\,, \qquad B = \kappa^2(\omega - \bar{\omega})\,. \tag{4.26}
$$

After eliminating the auxiliary fields, the bosonic action (4.24) becomes

$$
S = \int \mathrm{d}^4x\, e^{-1}\Big\{\frac{1}{2}\mathcal{R} - g_{i\underline{i}}\,\nabla^a Y^i\,\nabla_a\bar{Y}^{\underline{i}} - \frac{1}{4}\Big(\mathrm{e}^{2\varphi}\,(\nabla a)^2 + (\nabla\varphi)^2\Big) + \\
+ \frac{1}{\kappa^2}\Big(1 - \sqrt{-\det(\eta_{ab} + \kappa\,\mathrm{e}^{-\varphi/2}F_{ab})}\Big) + \frac{1}{4}\,a\,F^{ab}\tilde{F}_{ab}\Big\}\,. \tag{4.27}
$$

5. Photino dynamics in flat space

Our discussion of the fermionic dynamics in $\mathcal{N} = 1$ supersymmetric nonlinear electrodynamics will be restricted to the case of flat global superspace. Here the action takes the form

$$
S[W,\bar{W}] = \frac{1}{4}\int \mathrm{d}^6z\, W^2 + \frac{1}{4}\int \mathrm{d}^6\bar{z}\, \bar{W}^2 + \frac{1}{4}\int \mathrm{d}^8z\, W^2\,\bar{W}^2\,\Lambda(\omega,\bar{\omega})\,, \tag{5.1}
$$

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where

$$\omega \equiv \frac{1}{8} D^2 W^2 \,. \tag{5.2}$$

If $\Lambda(\omega, \bar{\omega})$ is a solution of the equation (4.4), then the above action obeys the self-duality equation [8, 9, 14]

$$\mathrm{Im} \int \mathrm{d}^6 z \left\{ W^2 + M^2 \right\} = 0 \,, \qquad \frac{\mathrm{i}}{2} M_\alpha = \frac{\delta}{\delta W^\alpha} S[W, \bar{W}] \,. \tag{5.3}$$

The action (5.1) can be seen to be invariant under a discrete chiral transformation

$$W_\alpha(x, \theta) \;\longrightarrow\; W_\alpha(x, -\theta) \,, \tag{5.4}$$

which leaves the fermionic fields invariant,

$$\psi_\alpha(x) = W_\alpha| \;\longrightarrow\; \psi_\alpha(x) \,, \tag{5.5}$$

whilst changing the bosonic fields as follows:

$$F_{\alpha\beta}(x) = \frac{1}{2\mathrm{i}} D_{(\alpha} W_{\beta)}| \;\longrightarrow\; -F_{\alpha\beta}(x) \,, \qquad D(x) = -\frac{1}{2} D^\alpha W_\alpha| \;\longrightarrow\; -D(x) \tag{5.6}$$

This symmetry implies that the component action contains only even powers of the bosonic fields. It is therefore consistent, when discussing the component structure, to restrict our attention to the purely fermionic sector specified by

$$D_\alpha W_\beta| = 0 \,. \tag{5.7}$$

Let $S[\psi, \bar{\psi}]$ be the fermionic action that follows from (5.1) upon switching off all the bosonic fields. It turns out that $S[\psi, \bar{\psi}]$ obeys a functional equation which is induced by the self-duality (5.3).

The self-duality equation (5.3) must hold for an arbitrary chiral spinor $W_\alpha(z)$ and its conjugate $\bar{W}_{\dot{\alpha}}(z)$. This means that the spinors W_α and $\bar{W}_{\dot{\alpha}}$ are chosen in (5.3) to satisfy only the chirality constraints $\bar{D}_{\dot{\alpha}} W_\alpha = 0$ and $D_\alpha \bar{W}_{\dot{\alpha}} = 0$, but not the Bianchi identity

$$D^\alpha W_\alpha = \bar{D}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \,. \tag{5.8}$$

Thus W_α now contains two independent fermionic components

$$\psi_\alpha(x) = W_\alpha| \,, \qquad \rho_\alpha(x) = -\frac{1}{4} D^2 W_\alpha| \,. \tag{5.9}$$

Let $\hat{S} \equiv S[\psi, \bar{\psi}, \rho, \bar{\rho}]$ be the component action that follows from (5.1) upon relaxing the Bianchi identity and restricting to the fermionic sector (5.7). Then, the self-duality equation (5.3) reduces to

$$\mathrm{Im} \int \mathrm{d}^4 x \left\{ \psi^\alpha \rho_\alpha + 4 \, \frac{\delta \hat{S}}{\delta \psi^\alpha} \, \frac{\delta \hat{S}}{\delta \rho_\alpha} \right\} = 0 \,. \tag{5.10}$$

The genuine fermionic action, $S[\psi, \bar{\psi}]$, is obtained from the self-dual action $\hat{S}$ by imposing the "fermionic Bianchi identities" $\rho_\alpha = -\mathrm{i}\, (\sigma^b \partial_b \bar{\psi})_\alpha$ and $\bar{\rho}_{\dot{\alpha}} = \mathrm{i}\, (\partial_b \psi \sigma^b)_{\dot{\alpha}}$,

$$S[\psi, \bar{\psi}] = S[\psi, \bar{\psi}, \rho, \bar{\rho}] \Big|_{\rho = -\mathrm{i}\, (\sigma^b \partial_b \bar{\psi})} \,. \tag{5.11}$$

A short calculation leads to the fermionic action

$$
\begin{aligned}
S[\psi,\bar\psi] = \int \mathrm{d}^4x \Big\{ & -\frac{1}{2}\langle u+\bar u\rangle + \Big(\langle u\rangle\langle\bar u\rangle - \frac{1}{4}(\partial^a\psi^2)(\partial_a\bar\psi^2)\Big)\Lambda(0,0) + \\
& + \Big(\langle u\rangle^2\langle\bar u\rangle + \frac{1}{2}(\bar\psi^2\Box\psi^2)\Big)\Lambda_\omega(0,0) + \Big(\langle u\rangle\langle\bar u\rangle^2 + \frac{1}{2}(\psi^2\Box\bar\psi^2)\Big)\Lambda_{\bar\omega}(0,0) + \\
& + \Big(\langle u\rangle^2\langle\bar u\rangle^2 - \frac{1}{2}(\partial^a\psi^2)(\partial_a\bar\psi^2)\langle u\rangle\langle\bar u\rangle + \frac{1}{16}\psi^2\bar\psi^2(\Box\psi^2)(\Box\bar\psi^2)\Big)\Lambda_{\omega\bar\omega}(0,0) + \\
& + \frac{3}{8}(\bar\psi^2\Box\psi^2)\langle u\rangle^2\,\Lambda_{\omega\omega}(0,0) + \frac{3}{8}(\psi^2\Box\bar\psi^2)\langle\bar u\rangle^2\,\Lambda_{\bar\omega\bar\omega}(0,0)\Big\}\,. \qquad (5.12)
\end{aligned}
$$

Here we have introduced the following 4×4 matrices:

$$
u_a{}^b = \mathrm{i}\,\psi\sigma^b\partial_a\bar\psi\,, \qquad \bar u_a{}^b = -\mathrm{i}\,(\partial_a\psi)\sigma^b\bar\psi\,, \qquad (5.13)
$$

as well as made use of the useful compact notation

$$
\langle F\rangle \equiv \mathrm{tr}\,F = F_a{}^a\,, \qquad (5.14)
$$

for an arbitrary 4×4 matrix $F = (F_a{}^b)$.

The fermionic action obtained involves several constant parameters associated with the function $\Lambda(\omega,\bar\omega)$ that enters the original supersymmetric action. However, not all of these parameters are independent since $\Lambda(\omega,\bar\omega)$ must be a solution to the self-duality equation (4.4). This restriction proves to imply

$$
\Lambda_\omega(0,0) = \Lambda_{\bar\omega}(0,0) = -\Lambda^2(0,0)\,, \qquad \Lambda_{\omega\omega}(0,0) = \Lambda_{\bar\omega\bar\omega}(0,0) = 2\Lambda^3(0,0)\,. \qquad (5.15)
$$

The self-duality equation imposes no restrictions on $\Lambda(0,0)$ and $\Lambda_{\omega\bar\omega}(0,0)$. For later convenience, we represent

$$
\Lambda(0,0) = \frac{\kappa^2}{2}\,, \qquad \Lambda_{\omega\bar\omega}(0,0) = \frac{\kappa^6}{8}(\mu+3)\,. \qquad (5.16)
$$

6. Relation to the Akulov-Volkov action

Looking at the fermionic action (5.12), it is hardly possible to imagine that it is related somehow to the Akulov-Volkov action (A.1), which describes Goldstino dynamics [28, 29] and which can be represented in the form

$$
\begin{aligned}
S_{\mathrm{AV}}[\lambda,\bar\lambda] = -\frac{1}{2}\int \mathrm{d}^4x \Big\{ & \langle v+\bar v\rangle + \frac{\kappa^2}{2}\Big(\langle v\rangle\langle\bar v\rangle - \langle v\bar v\rangle\Big) + \\
& + \frac{\kappa^4}{16}\Big(\langle v^2\bar v\rangle - \langle v\rangle\langle v\bar v\rangle - \frac{1}{2}\langle v^2\rangle\langle\bar v\rangle + \frac{1}{2}\langle v\rangle^2\langle\bar v\rangle + \text{c.c.}\Big)\Big\}\,, \quad (6.1)
\end{aligned}
$$

see appendix A for more details. Here

$$
v_a{}^b = \mathrm{i}\,\lambda\sigma^b\partial_a\bar\lambda\,, \qquad \bar v_a{}^b = -\mathrm{i}\,(\partial_a\lambda)\sigma^b\bar\lambda\,. \qquad (6.2)
$$

Nevertheless, the two fermionic theories turn out to be closely related in the following sense. There exists a nonlinear field redefinition, $(\psi_\alpha, \bar{\psi}^{\dot{\alpha}}) \rightarrow (\lambda_\alpha, \bar{\lambda}^{\dot{\alpha}})$, that eliminates all the higher derivative terms in (5.12) and brings this action to a one-parameter deformation of the AV action. The two theories coincide, modulo such a field redefinition, under the choice

$$\Lambda_{\omega\bar{\omega}}(0,0) = \frac{3}{8}\kappa^6 = 3\Lambda^3(0,0) \qquad \Longleftrightarrow \qquad \mu = 0\,, \tag{6.3}$$

which occurs, in particular, in the case of the supersymmetric Born-Infeld action [10–12]

$$\begin{aligned} S_{\text{SBI}} &= \frac{1}{4}\int \mathrm{d}^6z\, W^2 + \frac{1}{4}\int \mathrm{d}^6\bar{z}\, \bar{W}^2 + \frac{\kappa^2}{4}\int \mathrm{d}^8z\, \frac{W^2\,\bar{W}^2}{1+\frac{1}{2}\,A + \sqrt{1+A+\frac{1}{4}\,B^2}}\,, \\ A &= \kappa^2\,(\omega+\bar{\omega})\,, \qquad B = \kappa^2\,(\omega-\bar{\omega})\,, \qquad \omega = \frac{1}{8}D^2W^2\,. \end{aligned} \tag{6.4}$$

This section is devoted to the proof of the above statement.

We begin looking for a field redefinition by first noting that the leading order terms must match, $\psi_\alpha = \lambda_\alpha + O(\kappa^2)$. Next, to third-order in fields the general form of the redefinition can be written as

$$\psi_\alpha = \lambda_\alpha\Big\{1 + \frac{\kappa^2}{2}\alpha_1\langle v\rangle + \frac{\kappa^2}{2}\alpha_2\langle\bar{v}\rangle\Big\} + \frac{\mathrm{i}\kappa^2}{2}\alpha_3(\sigma^a\bar{\lambda})_\alpha(\partial_a\lambda^2) + O(\kappa^4)\,, \tag{6.5}$$

where the constant coefficients $\alpha_1, \alpha_2, \alpha_3$ can be chosen to be real. Substituting (6.5) into (5.12) gives

$$\begin{aligned} S[\psi,\bar{\psi}] = -\frac{1}{2}\int \mathrm{d}^4x \Big\{ &\langle v+\bar{v}\rangle + \kappa^2\alpha_1(\langle v\rangle^2 + \langle\bar{v}\rangle^2) + 2\kappa^2\Big(\alpha_2+\alpha_3-\frac{1}{2}\Big)\langle v\rangle\langle\bar{v}\rangle - \\ &- 2\kappa^2\alpha_3\langle v\bar{v}\rangle - \kappa^2(\alpha_3 - \frac{1}{4})(\partial^a\lambda^2)(\partial_a\bar{\lambda}^2)\Big\} + O(\kappa^4)\,. \end{aligned} \tag{6.6}$$

The requirement that the transformed action match with the AV action (6.1), uniquely fixes the coefficients $\alpha_1 = 0$, $\alpha_2 = \frac{1}{2}$, $\alpha_3 = \frac{1}{4}$. A similar calculation at fifth-order also allows us to match the AV action to this order. However, this calculation proves to be extremely tedious, as there exist many more admissible structures that can contribute to the field redefinition under consideration. Unlike the third-order case, not all coefficients are uniquely fixed — we are left with three free parameters, β_1, β_2, β_3. At the highest-order, this is again the case, and we gain another free parameter, γ. However, at this order, even with this freedom in the redefinition, it is impossible to match the AV action unless a restriction is placed on the type of model we are investigating, i.e. we must choose a particular value for $\Lambda_{\omega\bar{\omega}}(0,0)$.

With the following field redefinition

$$\begin{aligned} \psi_\alpha = \lambda_\alpha\Big\{1 + \frac{\kappa^2}{4}\langle\bar{v}\rangle + \frac{\kappa^4}{4}\Big(&\beta_1\langle v\rangle\langle\bar{v}\rangle + \beta_2\langle\bar{v}\rangle^2 + \Big(2\beta_3 - \frac{1}{4}\Big)\langle v\bar{v}\rangle - \\ &- \frac{1}{4}\langle\bar{v}^2\rangle + \beta_3(\partial^a\lambda^2)(\partial_a\bar{\lambda}^2) + \frac{1}{16}(\bar{\lambda}^2\Box\lambda^2)\Big) + \end{aligned}$$

$$
\begin{aligned}
&+\frac{\kappa^6}{64}\bigg((3\mu+1+4(\beta_1-2\beta_3-2\gamma))\langle v\rangle\langle\bar v\rangle^2-\\
&\qquad-2(\mu-2\beta_1-2\beta_2)\langle\bar v\rangle(\partial^a\lambda^2)(\partial_a\bar\lambda^2)+\\
&\qquad+\Big(\mu-\frac14-2\beta_1+4\beta_3\Big)\langle v\rangle(\bar\lambda^2\Box\lambda^2)+8(\beta_1+\beta_2-\beta_3)\langle v\bar v^2\rangle\bigg)\bigg\}+\\
&+\frac{\mathrm{i}}{8}\kappa^2(\sigma^a\bar\lambda)_\alpha(\partial_a\lambda^2)\Big\{1+\frac{\kappa^2}{2}(1-4(\beta_1+\beta_2+\beta_3))\langle v\rangle+\\
&\qquad+2\kappa^2\beta_3\langle\bar v\rangle+\kappa^4\gamma\langle v\rangle\langle\bar v\rangle\Big\}\,,
\end{aligned}
\tag{6.7}
$$

the transformed action is

$$
S[\psi,\bar\psi]=S_{\rm AV}[\lambda,\bar\lambda]+\frac{\kappa^6}{32}\,\mu\int \mathrm{d}^4x\,\langle v^2\bar v^2\rangle\,. \tag{6.8}
$$

We see that for the action (5.12), in conjunction with (5.15) and (5.16), is equivalent to the AV action (6.1) if eq. (6.3) holds. In particular, the supersymmetric Born-Infeld action (6.4) has this property.

Our field redefinition (6.7) involves four free parameters, β_1,β_2,β_3 and γ, which do not show up in the transformed action (6.8). This means that these parameters correspond to some symmetries of the original theory (5.12). Indeed, if we modify the field redefinition (6.7) by varying any of the parameters, the action is not affected.

7. Discussion

At first glance, the existence of the field redefinition (6.7) that turns the action (5.12) into (6.8), looks absolutely fantastic and unpredictable. However, it has a solid theoretical justification in one special case of self-dual supersymmetric electrodynamics (5.1) — the $\mathcal{N}=1$ supersymmetric Born-Infeld action (6.4). This action is known to describe the Goldstone-Maxwell multiplet for spontaneous partial supersymmetry breaking $\mathcal{N}=2\to\mathcal{N}=1$ [11, 12]. As a consequence, its purely fermionic sector

$$
\begin{aligned}
S_{\rm BG}[\psi,\bar\psi]=\int\mathrm{d}^4x\Big\{&-\frac12\langle u+\bar u\rangle+\frac{\kappa^2}{2}\Big(\langle u\rangle\langle\bar u\rangle-\frac14(\partial^a\psi^2)(\partial_a\bar\psi^2)\Big)-\\
&-\frac{\kappa^4}{4}\Big(\langle u\rangle^2\langle\bar u\rangle+\langle u\rangle\langle\bar u\rangle^2+\frac12(\bar\psi^2\Box\psi^2)+\frac12(\psi^2\Box\bar\psi^2)\Big)+\\
&+\frac{3\kappa^6}{32}\Big(4\langle u\rangle^2\langle\bar u\rangle^2-2(\partial^a\psi^2)(\partial_a\bar\psi^2)\langle u\rangle\langle\bar u\rangle+\frac14\psi^2\bar\psi^2(\Box\psi^2)(\Box\bar\psi^2)+\\
&\qquad+(\bar\psi^2\Box\psi^2)\langle u\rangle^2+(\psi^2\Box\bar\psi^2)\langle\bar u\rangle^2\Big)\Big\}\,,
\end{aligned}
\tag{7.1}
$$

which follows from (5.12), turns out to describe spontaneous breakdown of $\mathcal{N}=1$ supersymmetry [35]. This Goldstino action clearly does not coincide with the standard Goldstino action (A.1) or, equivalently, with (6.1). Universality of the Goldstino dynamics, on the other hand, implies that the two Goldstino actions, (A.1) and (7.1), should be related to each other. It was therefore conjectured in [35] that the actions (A.1) and (7.1) are related by a nontrivial field redefinition. Moreover, guided by considerations of nonlinearly realized supersymmetry, the authors of [35] proposed a nice scheme for constructing such a field redefinition and also confirmed it to order κ^2 (see the first line in (6.7)). Pushing their scheme to higher orders seems to give the redefinition (6.7) with all the parameters fixed as follows: $\beta_1 = \frac{1}{16}$, $\beta_2 = 0$, $\beta_3 = \frac{1}{32}$ and $\gamma = 0$. We have checked the correspondence to order κ^4.

The field redefinition (6.7) corresponds to the purely fermionic sector of the globally supersymmetric theory (5.1). In the case when both bosonic and fermionic fields are present, as well as in the presence of supergravity – the case we analyzed in section 4, there should exist an extension of (6.7) that, at least, eliminates all higher derivative terms from the component action. But here our brute-force approach becomes extremely cumbersome and tedious to follow (even the fermionic case was quite a pain). We believe that there should be a more efficient approach to construct such field redefinitions. Unfortunately, it is beyond our grasp at the moment. It is worth pointing out that the issue of constructing nonlinear field redefinitions that eliminate higher derivatives, is quite typical in supersymmetric field theories. It naturally occurs when studying low-energy effective actions in extended super Yang-Mills theories [36, 37].

In conclusion, we would like to make a final comment regarding the supersymmetric Born-Infeld action (6.4). In the purely bosonic sector, this theory reduces, upon elimination of the auxiliary field, to the Born-Infeld action

$$S_{\rm BI} = \frac{1}{\kappa^2} \int {\rm d}^4x \left(1 - \sqrt{-\det(\eta_{ab} + \kappa\, F_{ab})}\right), \tag{7.2}$$

compare with (4.27). In the purely fermionic sector, it reduces, upon implementing the field redefinition (6.7) with $\mu = 0$, to the Akulov-Volkov action (A.1). In the general case, it should describe, upon implementing a nonlinear field redefinition, the space-time filling D3-brane in a special gauge for kappa-symmetry, see [38, 39] and references therein. Such a gauge differs from the one chosen in [38].

The supersymmetric Born-Infeld action (6.4) is just a special representative in the family of self-dual models (5.1), with $\Lambda(\omega, \bar{\omega})$ a solution of the differential equation (4.4). But it is only the action (6.4) which describes the partial supersymmetry breaking $\mathcal{N} = 2 \to \mathcal{N} = 1$. At the component level, however, the purely fermionic action has been shown to be equivalent to the Goldstino action (A.1) under the mild condition (6.3), which holds for infinitely many members of the family, including the supersymmetric Born-Infeld action (6.4). In this sense, all such models contain information about spontaneous supersymmetry breaking.

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A. The Akulov-Volkov action

The Akulov-Volkov (AV) action for the Goldstino [28] is[6]

$$S_{\rm AV}[\lambda,\bar\lambda] = \frac{2}{\kappa^2}\int {\rm d}^4x \Big\{1-\det\Xi\Big\}\,, \tag{A.1}$$

where

$$\Xi_a{}^b = \delta_a{}^b + \frac{\kappa^2}{4}\Big({\rm i}\,\lambda\sigma^b\partial_a\bar\lambda - {\rm i}\,(\partial_a\lambda)\sigma^b\bar\lambda\Big) \equiv \delta_a{}^b + \frac{\kappa^2}{4}(v+\bar v)_a{}^b\,. \tag{A.2}$$

With the notation (5.14), the Goldstino action can explicitly be rewritten as a polynomial in v and $\bar v$:

$$\begin{aligned} S_{\rm AV}[\lambda,\bar\lambda] \;=\; -\frac{1}{2}\int {\rm d}^4x\Big\{ &\langle v+\bar v\rangle + \frac{\kappa^2}{8}\Big(\langle v+\bar v\rangle^2 - \langle (v+\bar v)^2\rangle\Big) + \\ &+\frac{\kappa^4}{16}\Big(\langle v^2\bar v\rangle - \langle v\rangle\langle v\bar v\rangle - \frac12\langle v^2\rangle\langle\bar v\rangle + \frac12\langle v\rangle^2\langle\bar v\rangle + {\rm c.c.}\Big) - \\ &-\frac{\kappa^6}{64}\Big(\langle v^2\bar v^2\rangle + \frac12\langle v\bar v v\bar v\rangle - \Big[\langle v\rangle\langle v\bar v^2\rangle - \frac14\langle v\rangle^2\langle\bar v^2\rangle + {\rm c.c.}\Big] + \\ &+\langle v\rangle\langle\bar v\rangle\langle v\bar v\rangle - \frac12\langle v\bar v\rangle^2 - \frac14\langle v^2\rangle\langle\bar v^2\rangle - \frac14\langle v\rangle^2\langle\bar v\rangle^2\Big)\Big\}\,. \end{aligned} \tag{A.3}$$

The fourth-order terms can be simplified slightly:

$$\frac14\int {\rm d}^4x\,\Big(\langle v+\bar v\rangle^2 - \langle (v+\bar v)^2\rangle\Big) = \int {\rm d}^4x\,\Big(\langle v\rangle\langle\bar v\rangle - \langle v\bar v\rangle\Big)\,. \tag{A.4}$$

Regarding the eighth-order terms, the situation is more dramatic. Using the (easily verified) identities

$$\begin{aligned} \langle v^2\bar v^2\rangle \;&=\; \Big(\langle v\rangle\langle v\bar v^2\rangle - \frac12\langle v\rangle^2\langle\bar v^2\rangle + {\rm c.c.}\Big) + \langle v\bar v\rangle\Big(\langle v\bar v\rangle - \langle v\rangle\langle\bar v\rangle\Big)\,, \\ 2\langle v\bar v v\bar v\rangle \;&=\; \langle v^2\rangle\langle\bar v^2\rangle - 2\langle v\bar v\rangle^2 + \Big(\langle v\rangle^2\langle\bar v^2\rangle + {\rm c.c.}\Big) + \langle v\rangle^2\langle\bar v\rangle^2\,, \end{aligned} \tag{A.5}$$

one can check that the eighth-order terms in (A.3) completely cancel out! This result may seem strange, since the eighth-order terms in the AV action are, to the best of our knowledge, explicitly retained in all relevant publications, starting with the classic papers by Akulov and Volkov [28, 29] and continuing today, e.g. [40] (see, however [41] where it is demonstrated that the energy-momentum tensor for the AV model does not contain any eighth-order terms). Therefore we will give another, purely algebraic and quite elementary, proof.

The whole contribution from the eighth-order terms in the integrand in (A.3) can be shown to be proportional to

$$\varepsilon_{abcd}\,\varepsilon^{klmn}\,v_k{}^a v_l{}^b \bar v_m{}^c \bar v_n{}^d = \lambda^2\bar\lambda^2\,\varepsilon_{abcd}\,\varepsilon^{klmn}\,\Big(\partial_k\bar\lambda\,\tilde\sigma^{ab}\,\partial_l\bar\lambda\Big)\Big(\partial_m\lambda\,\sigma^{cd}\,\partial_n\lambda\Big)\,. \tag{A.6}$$

[6]Note that the normalization factor used here differs from that of $1/2\kappa^2$ usually found in the literature. This is in order to match up with the coupling constant in the bosonic sector of the supersymmetric Born-Infeld action (7.2).

Using the well-known property of the sigma-matrices,

$$\frac{1}{2}\,\varepsilon_{abcd}\,\sigma^{cd} = -\mathrm{i}\,\sigma_{ab}\,, \quad \frac{1}{2}\,\varepsilon_{abcd}\,\tilde{\sigma}^{cd} = \mathrm{i}\,\tilde{\sigma}_{ab}\,, \quad \longrightarrow \quad (\sigma^{ab})_\alpha{}^\beta\,(\tilde{\sigma}_{ab})^{\dot\alpha}{}_{\dot\beta} = 0\,, \tag{A.7}$$

we see that the whole contribution under consideration vanishes. As a result, the AV action takes the form (6.1).

B. Old minimal supergravity: alternative realization

Here we consider an alternative realization of the model for old minimal supergravity (2.14) that is obtained by employing a variant superfield representation [42] of the form

$$\Sigma^3 = -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4R)\,P\,, \qquad \bar{P} = P\,, \tag{B.1}$$

with P an unconstrained real scalar superfield. It follows from (2.15) that the super-Weyl transformation of P should be

$$P \;\rightarrow\; \mathrm{e}^{-\sigma-\bar\sigma}\,P\,, \tag{B.2}$$

compare with (2.9). In this approach, the covariantly chiral scalar Σ occurs not as one of the dynamical fields, but instead as a gauge-invariant field strength associated with the following gauge freedom:

$$\delta P = L\,, \qquad (\bar{\mathcal{D}}^2 - 4R)\,L = 0\,. \tag{B.3}$$

Unlike the vector multiplet case, the component vector field contained in P should now be interpreted as a gauge three-form, as the action of (B.3) on this field coincides with a three-form gauge transformation [42]. The gauge-invariant four-form field strength, which is associated with the three-form, appears as one of the two auxiliary fields contained in Σ^3. The latter implies, in fact, that a supersymmetric cosmological term [25, 26] becomes real,

$$\mathrm{Im}\int \mathrm{d}^8 z\,\frac{E^{-1}}{R}\,\Sigma^3 = 0\,. \tag{B.4}$$

An interesting feature of this realization for old minimal supergravity is that it possesses solutions with a non-vanishing cosmological constant, without the need to explicitly add a supersymmetric cosmological term, $\mu\int \mathrm{d}^8 z\,(ER)^{-1}\,\Sigma^3$, to the supergravity action (2.14). Another intriguing property of this formulation is that, unlike the standard realization, it allows a simple construction of massive off-shell supergravitons [43].

If one only insists on the super-Weyl invariance (B.2) without requiring the gauge symmetry (B.3), a more general action, involving the naked prepotential P, can be considered

$$\begin{aligned} S &= \int \mathrm{d}^8 z\,E^{-1}\,P\,\mathcal{F}\Big(\frac{\bar\Sigma\,\Sigma}{P}\Big) + \Big\{g\int \mathrm{d}^8 z\,\frac{E^{-1}}{R} W^2 + \mathrm{c.c.}\Big\}\,, \\ W_\alpha &= -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha\,\ln P\,, \end{aligned} \tag{B.5}$$

with a real function $\mathcal{F}(x)$ and a constant parameter g. In general, such an action describes couplings of supergravity to supersymmetric matter, see e.g. [44]. It corresponds to pure supergravity only if $g = 0$ and $\mathcal{F}(x)$ is a linear function, $\mathcal{F}(x) = -3x + \mu$.

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